Classical Non-Local conserved charges in String Theory

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We construct a conserved non local charge in $AdS_5 \times S_5$ string theory.

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String theory is the strongest candidate to describe all interactions[1]. However, there is no crucial experiment giving full support of the theory which is still in need of a stronger basis for a full theory of nature. As happens in several field theories, non perturbative results may give important clues to the behaviour of the theory in particularly difficult situations. Several results have already been obtained from the idea of duality [2]. Recently some new features connected with the high dimensionality of strings have led to further insight into the structure of the socalled brane cosmology [3, 4, 5].

More recently, some authors are pursuing higher conservation laws [6, 7, 8], which proved of great help in theories of lower dimensionality [9, 10, 11, 12]. In case we can use higher conservation laws in a way similar to the one used in two dimensional space time, it is possible that further constraints in the dynamical behaviour of strings can be imposed and one can gather information based on more general grounds to be compared with observations. As an example, strong nonperturbative insight about gravity can be obtained from the holographic principle [13].

We begin our problem with a set of currents defined in $AdS_5 \times S_5$ space described by the coset $PSU(2,2|4)/SO(4,1)\times SO(5)$ [14]. The underlying string theoryhas been described accordingly [15]. The algebra $psu(2,2|4)$ has, under the discrete group Z_4, a discrete decomposition $\mathcal{H} = \oplus_{i=0}^{3}\mathcal{H}_i$, described by

$$\begin{array}{cc} t_\alpha \in \mathcal{H}_1 \quad , & t_{\underline{a}} \in \mathcal{H}_2 \quad , \\ t_{\hat\alpha} \in \mathcal{H}_3 \quad , & t_{\underline{[ab]}} \in \mathcal{H}_0 \end{array} \tag{1}$$

where $\underline{a}$ are indices parametrizing $AdS_5 \times S_5$, α and $\hat\alpha$ are the superspace connections. The non vanishing structure constants are well known [8].

In terms of the supercoset valued filed $g(x,\theta,\hat\theta)$ we can define algebra valued currents $\mathbf{J} = g^{-1}dg$, which in turn are can be decomposed according to (1). The resulting currents have been shown to obey the relations [8]

$$\begin{aligned} \partial_\mu J_1^\mu + [J_{0\mu}, J_1^\mu] &= \varepsilon^{\mu\nu}\left[J_{2\mu}, J_{3\nu}\right] \quad , \\ \partial_\mu J_2^\mu + [J_{0\mu}, J_2^\mu] &= \frac{1}{2}\varepsilon^{\mu\nu}\left([J_{3\mu}, J_{3\nu}] - [J_{1\mu}, J_{1\nu}]\right) \quad , \\ \partial_\mu J_3^\mu + [J_{0\mu}, J_3^\mu] &= -\varepsilon^{\mu\nu}\left[J_{2\mu}, J_{1\nu}\right] \quad , \end{aligned} \tag{2}$$

and

$$\begin{aligned} \varepsilon^{\mu\nu}\left(\partial_\mu J_{1\nu} + [J_{0\mu}, J_{1\nu}]\right) &= -\varepsilon^{\mu\nu}\left[J_{2\mu}, J_{3\nu}\right] \quad , \\ \varepsilon^{\mu\nu}\left(\partial_\mu J_{2\nu} + [J_{0\mu}, J_{2\nu}]\right) &= -\frac{1}{2}\varepsilon^{\mu\nu}\left([J_{3\mu}, J_{3\nu}] + [J_{1\mu}, J_{1\nu}]\right) \quad , \\ \varepsilon^{\mu\nu}\left(\partial_\mu J_{3\nu} + [J_{0\mu}, J_{3\nu}]\right) &= -\varepsilon^{\mu\nu}\left[J_{2\mu}, J_{1\nu}\right] \quad , \end{aligned} \tag{3}$$

in the underlying Minkowski space. Such relations keep some similarity with zero curvature conditions, well known in integrable models [9] and nonlinear sigma models [10, 11], implying, upon quantization, severe constraints upon the S-matrix elements [12].

Here, we try a constructive approach. First, we neglect terms related to gauge-field valued elements of the algebra, which simplifies the discussion. In fact, the relevant currents transform nontrivially under gauge transformations,

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which is mirrored in the fact that the derivatives in the conservation laws are covariant derivatives of the form $\partial + [\mathbf{J}_0,]$. We also notice that some commutators are gauge valued, such as $[\mathbf{J}_2, \mathbf{J}_2]$, or $[\mathbf{J}_1, \mathbf{J}_3]$. In expression (2), there are several conservation laws inbuilt and we are going to construct one of them. We claim that a non local conserved charge should be described in terms of a gauge dressing of a combination of the following building blocks:

$$
\begin{aligned}
Q^{(1)} &= 2\int J_3^0(t,x_1)dx_1 \quad ,\\
Q^{(2)} &= -\int J_1^0(t,x_1)\epsilon(x_1-x_2)J_2^0(t,x_2)dx_1dx_2 - \int J_2^0(t,x_1)\epsilon(x_1-x_2)J_1^0(t,x_2)dx_1dx_2 \quad ,\\
Q^{(3)} &= \frac{1}{2}\int J_1^0(t,x_1)\epsilon(x_1-x_2)J_1^0(t,x_2)\epsilon(x_2-x_3)J_1^0(t,x_3)dx_1dx_2dx_3\\
&- \frac{1}{2}\int J_3^0(t,x_1)\epsilon(x_1-x_2)J_3^0(t,x_2)\epsilon(x_2-x_3)J_1^0(t,x_3)dx_1dx_2dx_3\\
&- \frac{1}{2}\int J_3^0(t,x_1)\epsilon(x_1-x_2)J_1^0(t,x_2)\epsilon(x_2-x_3)J_3^0(t,x_3)dx_1dx_2dx_3\\
&- \frac{1}{2}\int J_1^0(t,x_1)\epsilon(x_1-x_2)J_3^0(t,x_2)\epsilon(x_2-x_3)J_3^0(t,x_3)dx_1dx_2dx_3\\
&- \frac{1}{2}\int J_2^0(t,x_1)\epsilon(x_1-x_2)J_2^0(t,x_2)\epsilon(x_2-x_3)J_3^0(t,x_3)dx_1dx_2dx_3\\
&- \frac{1}{2}\int J_2^0(t,x_1)\epsilon(x_1-x_2)J_3^0(t,x_2)\epsilon(x_2-x_3)J_2^0(t,x_3)dx_1dx_2dx_3\\
&- \frac{1}{2}\int J_3^0(t,x_1)\epsilon(x_1-x_2)J_2^0(t,x_2)\epsilon(x_2-x_3)J_2^0(t,x_3)dx_1dx_2dx_3 \quad ,\\
Q^{(4)} &= \frac{1}{4}\sum\int J_1^0(x_1)\epsilon(x_1-x_2)J_1^0(x_2)\epsilon(x_2-x_3)J_2^0(x_3)\epsilon(x_3-x_4)J_3^0(x_4)dx_1dx_2dx_3dx_4\\
&+ \frac{1}{4}\sum\int J_1^0(x_1)\epsilon(x_1-x_2)J_2^0(x_2)\epsilon(x_2-x_3)J_2^0(x_3)\epsilon(x_3-x_4)J_2^0(x_4)dx_1dx_2dx_3dx_4\\
&- \frac{1}{4}\sum\int J_2^0(x_1)\epsilon(x_1-x_2)J_3^0(x_2)\epsilon(x_2-x_3)J_3^0(x_3)\epsilon(x_3-x_4)J_3^0(x_4)dx_1dx_2dx_3dx_4 \quad ,\\
&\quad ...
\end{aligned}
\tag{4}
$$

where the sum is over all orders of the indices of currents. The remaining terms are to be constructed taking into account the generic additive term

$$
Q_i^{(n)} = \pm\frac{1}{2^{n-2}}\sum\int\left(\prod_{k=1}^{n-1}J_{\alpha_k}^0(t,x_k)\epsilon(x_k-x_{k+1})dx_k\right)J_{\alpha_n}^0(t,x_n)dx_n, \tag{5}
$$

with the indices α_k satisfying the constraint equation

$$
\sum_{k=1}^{n}\alpha_k = 3 \qquad \mathrm{mod}\ 4. \tag{6}
$$

The sign has to be properly chosen in order to achieve conservation of the sum. The time derivative of the current can be exchanged by the space derivative, a commutator with a gauge-valued current and a nontrivial commutator. The space derivative is integrated by parts giving rise to a Dirac delta used to perform one integration, and leaving a lower order term in the integration variables, but with a nontrivial commutator. The commutator either cancels a similar one arising from a lower order derivative, or is gauge valued. Therefore, we claim that

$$
\partial_0 Q^{(n)} = \delta_{der}Q^{(n)} + \delta_{com}Q^{(n)} + \text{gauge terms} \tag{7}
$$

where the first term arises from the integration by parts of the space derivative of the current giving rise to a Dirac-delta term of the form of a commutator among two currents connected by the $\epsilon(x_k - x_{k+1})$ function, while the second term arises from the remaining term of the equation of motion, with exception of the gauge ($\mathbf{J}_0$) commutators. In the first term we leave aside the commutators which take values in the gauge sector, namely, $[\mathbf{J}_2, \mathbf{J}_2]$ and $[\mathbf{J}_1, \mathbf{J}_3]$, which are, together with the explicit gauge terms containing $\mathbf{J}_0$ left to the last term. Under these conditions, the definition of the charges leads us to the result

$$
\delta_{der}Q^{(n+1)} = -\delta_{com}Q^{(n)} \quad . \tag{8}
$$

Therefore, we

Claim: the nonlocal charge

$$Q_3 = \sum_{n=1}^{\infty} Q^{(n)} \tag{9}$$

is classically conserved up to a gauge dressing, defined substituting some currents by $\mathcal{H}_0$ valued elements.

We also claim that analogous charges, whose first term are either obtained from J_2^0 or from J_1^0 are also conserved.

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[1] M. Green, J. Schwarz and E. Witten *Superstring Theory*, Cambridge Univ. Pr. (1987), Cambridge Monographs On Mathematical Physics.

[2] J. Polchinski *Superstring Theory*, Cambridge University Press 1998.

[3] E. Witten *Nucl. Phys.* **B443** (1995) 85, hep-th/9503124.

[4] P. Horava and E. Witten *Nucl. Phys.* **B460** (1996) 506, **B475** (1996) 94.

[5] L. Randall and R. Sundrum *Phys. Rev. Lett.* **83** (1999) 3370, 4690.

[6] Iosif Bena, Joseph Polchinski, Radu Roiban *Phys. Rev.* **D69** (2004) 046002, hep-th/0305116; Bin Chen, Xiao-Jun Wang and Yong-Shi Wu *Phys. Lett.* **B591** (2004) 170 hep-th/0403004, *JHEP* **0402** (2004) 029, hep-th/0401016.

[7] L. Dolan, C. Nappi and E. Witten *JHEP* **0310** (2003) 017, hep-th/0308089.

[8] Brenno Carlini Vallilo, *JHEP* **0403** (2004) 037, hep-th/0307018.

[9] D. Gross, Neveu *Phys. Rev.* **D10** (1974)3235; E. Abdalla and A. Lima-Santos *Rev. Bras. Fis.* **12** (1982) 293.

[10] M. Lüscher *Nucl. Phys.* **B135** (1978) 1; M. Luscher and K. Pohlmeyer *Nucl. Phys.* **B137** (1978) 46.

[11] E. Abdalla, M. C. B. Abdalla and M. Gomes *Phys. Rev.***D23** (1981) 1800, **D25** (1982) 452, **D27** (1983) 825; E. Abdalla, M. Forger and M. Gomes *Nucl. Phys.* **B210** [FS6] (1982) 181; E. Abdalla, M. Forger and A. Lima-Santos *Nucl. Phys.* **B256** (1985) 145; E. Abdalla, M. C. B. Abdalla and M. Forger *Nucl. Phys.* **B297** (1988) 374; E. Abdalla and A. Lima-Santos, *Mod. Phys. Lett.* **A3** (1988) 310, *Phys. Rev.* **D29** (1984) 1851; E. Abdalla, M.C.B. Abdalla and A. Lima-Santos, *Phys. Lett.* **B140** (1984) 71-75, Erratum *B146* (1984) 457.

[12] E. Abdalla, M. C. B. Abdalla and K. D. Rothe, *Non perturbative methods in two dimensional quantum field theory*, World Scientific Publishing Company, 2001.

[13] Leonard Susskind *J. Math. Phys.* **36** (1995) 6377 hep-th/9409089; Gerard 't Hooft *Salamfest* 1993, 0284-296 (QCD161:C512:1993) gr-qc/9310026.

[14] R. R. Metsaev and A. A. Tseytlin *Nucl. Phys.* **B596** (2001) 185, hep-th/0009168.

[15] N. Berkovits *JHEP* **04** (2000) 018; N. Berkovits and O. Chandia *Nucl. Phys.* **B596** (2001) 185; B. Vallilo *JHEP* **0212** (2002) 042.